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M. Sc. (Sem. - IV) Examination September - 2025
Mathematics (Paper - PGMTH 403) Integral Equations

Time: 3 Hours] **[Total Marks: 70**

સૂચના : / Instructions

(1) ઉપરોક્ત દર્શાવેલ વિગતો ઉત્તરવહી પર અવશ્ય લખવી.
Fill up strictly the above details on your answer book.
(2) All questions are compulsory.
(3) Follow usual notations and conventions.
(4) Figures to the right indicate marks of the question.

Seat No.:

Student's Signature

Q. 1. Attempt any two: **(14)**

a. Show that the function $y(x) = (1+x^2)^{-3/2}$ is a solution of the Volterra integral equation $y(x) = \frac{1}{1+x^2} - \int_0^x \frac{t}{1+x^2}\, y(t)\, dt$.

b. State Leibnitz's rule for differentiation of a function under integral sign.
Evaluate $\frac{d}{dx}\left[\frac{x}{2} + \frac{x^2}{2}\int_0^1 k(x,t)y(t)\,dt\right]$, where
$k(x, t) = \begin{cases} t^2(1-x), & 0 \le t \le x; \\ x^2(1-t), & x \le t \le 1. \end{cases}$

c. Derive a formula to convert multiple integral into a single ordinary integral.

Q. 2. Attempt any two: **(14)**

a. Convert the initial value problem $y'' + A(x)y' + B(x)y = f(x),\ y(a) = y_0,\ y'(0) = y_1$ to an integral equation.

b. Derive the initial value problem corresponding to following integral equations:
(i) $y(x) = 1 + \int_0^x (x-t)y(t)\,dt$
(ii) $y(x) = 1 + \int_0^x y(t)\,dt$.

c. Explain the alternative method to convert an initial value problem into a Volterra integral equation.

Q. 3. Attempt any two: **(14)**

a. Convert the following boundary value problem into a Fredholm integral equation:
$\frac{d^2y}{dx^2} + \lambda y = 0$, with $y(0) = 0,\ y(l) = 0$.

b. Derive the boundary value problem corresponding to the integral equation
$y(x) = \frac{x(x-1)}{2} + \int_0^1 k(x, t)y(t)\,dt$ where
$k(x, t) = \begin{cases} (1-x)t^2, & 0 \le t \le x; \\ (1-t)x^2, & x \le t \le 1. \end{cases}$

c. Show that the integral equation $y(x) = \lambda \int_0^1 (3x-2)\, t\, y(t)\, dt$ has no eigen value.

Q. 4. Attempt any two: **(14)**

a. Discuss the method to solve integral equation $y(x) = f(x) + \lambda \int_a^b k(x, t)\, y(t)\, dt$ with separable kernel.

b. Define: Eigen values and eigen functions of homogeneous Fredholm integral equation of second kind with separable kernel. Solve: $y(x) = \lambda \int_0^1 e^{x+t} y(t)\, dt$.

c. Obtain the $y(x) = \lambda \int_0^1 (x \cosh t - t \cosh x)\, y(t)\, dt$ doesn't have real eigen values and eigen functions.

Q. 5. Attempt any two: **(14)**

a. Prove that the m^{th} iterated kernel $K_m(x, t)$ satisfies the relation

$K_m(x, t) = \int_a^b K_r(x, y)\, K_{m-r}(y, t)\, dy$,

where r is any positive integer less than m.

b. Solve $y(x) = \sin x + 2 \int_0^x e^{x-t}\, y(t)\, dt$ by successive approximation.

c. Define Resolvent kernel. Determine the resolvent kernel for the Fredholm integral equations having kernel $k(x, t) = (1+x)(1-t)$, $a = -1$, $b = 1$.